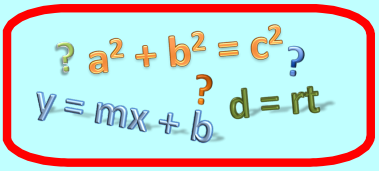
Math 115

Spring 2018

Lecture 13



System of linear equations in two Variables:

A linear eqn in two variables looks like $Ax + By = C$ or $y = mx + b$, or $y - y_1 = m(x - x_1)$

$3x - 5y = 8$, $y = 2x - 3$, $y + 4 = \frac{1}{2}(x - 3)$

$x = 3y + 5$, ...

System of linear eqns in 2 Variables is when we have more than 1 eqn.:

$\begin{cases} x + y = 4 \\ x - y = 8 \end{cases}$, $\begin{cases} 3x - 2y = -6 \\ y = \frac{3}{2}x + 5 \end{cases}$, $\begin{cases} \frac{1}{2}x - \frac{2}{3}y = 1 \\ \frac{1}{4}x + \frac{3}{5}y = -1 \end{cases}$

Math 125

$\begin{cases} x + y - z = 7 \\ 2x - 3y + z = -10 \\ 3x + 4y + 5z = -20 \end{cases}$

The Solution to a system of linear eqns in two variables is an ordered-pair (x, y) which satisfies both eqns.

Is $(-3,2)$ a Solution of

$$\begin{cases} x + y = -1 \\ 2x - 3y = -12 \end{cases} ?$$

$2x - 3y = -12$ ✓

$2(-3) - 3(2) = -12$

$-6 - 6 = -12$

$x + y = -1$ ✓

$-3 + 2 = -1$

$-1 = -1$

both eqns are satisfied, so $(-3,2)$ is a Soln.

Is $(5,-2)$ a Solution of

$$\begin{cases} 3x - y = 17 \\ 4x + 5y = -10 \end{cases} \quad ? \quad \checkmark \; \times$$

So $(5,-2)$ is not a Solution.

Checking Eqn ①:

$3(5) - (-2) = 17$

$15 + 2 = 17$ ✓

Checking Eqn ②:

$4(5) + 5(-2) = -10$

$20 - 10 = -10$

$10 = -10$ ×

When the system of equation has

(a) Exactly one Solution ⇒ System is Consistent. Equations are independent.

(b) infinitely Many Solutions ⇒ System is Consistent. Eqns are dependent.

(c) has no Solution ⇒ System is inconsistent. Equations are independent.

How to Solve System of linear equations in two Variables:

Math 115
1. Graphing
2. Substitution
3. Addition/Elimination

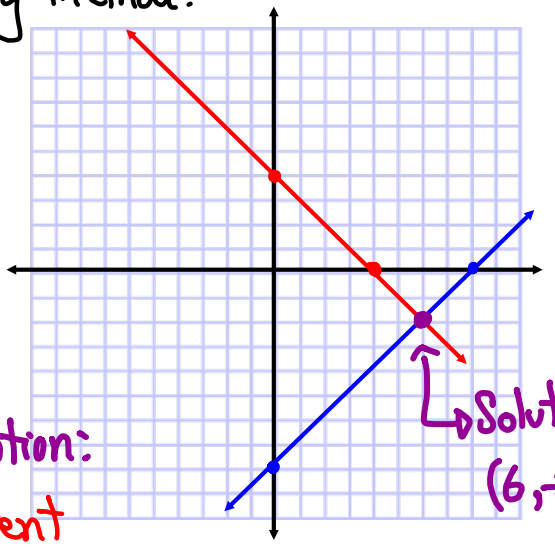
Math 125
4. Cramer's Rule
5. Matrix Method

Solve by Graphing Method:

$$\begin{cases} x+y=4 \\ x-y=8 \end{cases}$$

x	y
0	4
4	0

x	y
0	-8
8	0



Exactly One Solution:

System: Consistent

Equations: Independent.

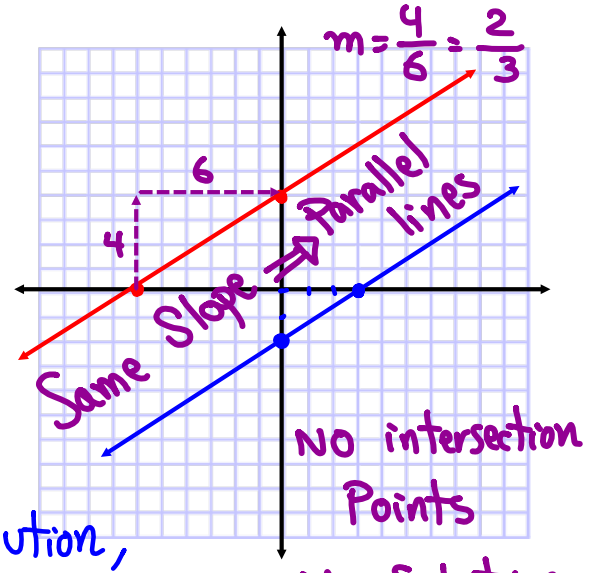
Solve by Graphing:

$$\begin{cases} 2x-3y=-12 \\ y=\frac{2}{3}x-2 \end{cases}$$

x	y
0	4
-6	0

$m=\frac{2}{3}$

Y-Int (0,-2)



Since there is no Solution,

System: Inconsistents

Equations: Independent eqns.

NO Solution

$\emptyset$

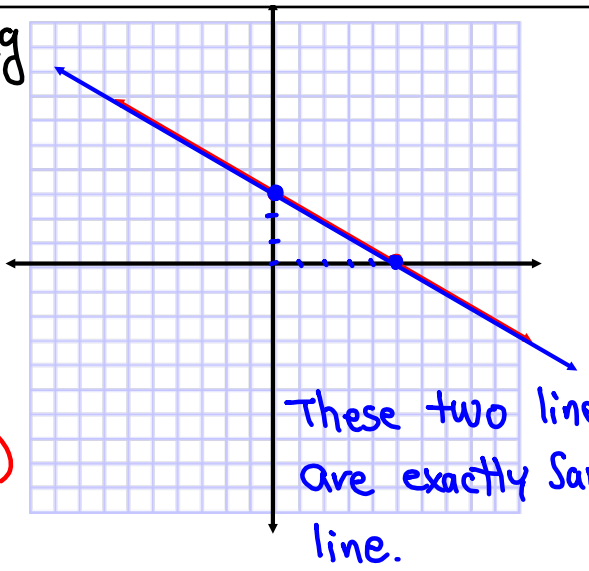
Solve by Graphing

$$\begin{cases} 3x + 5y = 15 \\ y = \frac{-3}{5}x + 3 \end{cases}$$

x	y
0	3
5	0

$m = \frac{-3}{5}$

Y-Int (0,3)



These two lines are exactly same line.

System: Consistent

Equations: Dependent

They have infinitely many points in Common.

Infinitely Many Solutions

Solve by Substitution

$$\begin{cases} 3x - y = 18 \\ y = x \end{cases} \rightarrow 3x - x = 18$$

$2x = 18$

$x = 9 \rightarrow y = 9$

Soln $(9, 9)$

System: Consistent

Eqns: Independent

Solve by Subs.:

$$\begin{cases} 2x + y = 15 \\ y = 7 - x \end{cases}$$

$2x + (7 - x) = 15$

$x + 7 = 15$

$x = 8$

$y = 7 - 8$

$y = -1$

System: Consistent

Eqns: Independent

Soln $(8, -1)$

Solve by Subs:

$$\begin{cases} 2x - y = 8 \\ y = 2x + 3 \end{cases}$$

$2x - (2x + 3) = 8$

$2x - 2x - 3 = 8$

$-3 = 8$

False

NO Solution

$\emptyset$

System: Inconsistent.

Eqns: Independent.

Solve by Subs method

$$\begin{cases} 6x - 2y = 4 \\ y = 3x - 2 \end{cases}$$

$$6x - 2(3x - 2) = 4$$

$$6x - 6x + 4 = 4$$

$$4 = 4$$

True

System: Consistent

Eqns: Dependent

Infinitely Many Soln.

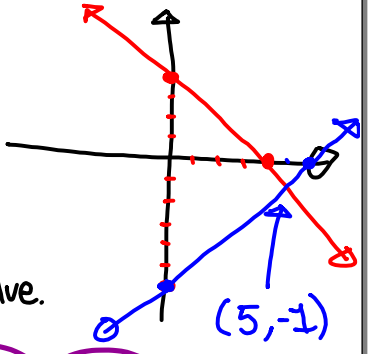
The sum of two numbers is 4.

Their difference is 6.

Find both numbers using system of linear eqns in two variables.

$$\begin{cases} x + y = 4 \\ x - y = 6 \end{cases}$$

use graphing method to solve.



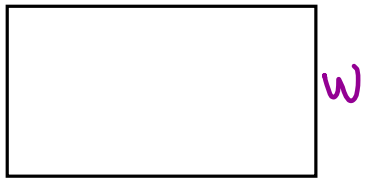
The numbers are 5 and -1.

The perimeter of a rectangular room is 42 ft. The length is 1 ft longer than 3 times its width use system of linear eqns in two variables to find its dimensions.

$P = 42$ ft

$$\begin{cases} 2L + 2W = 42 \\ L = 3W + 1 \end{cases}$$

Proceed by using Subs. method



$2(3W+1) + 2W = 42$

$6W + 2 + 2W = 42$

$8W = 40$

$W = 5$

$L = 3(5) + 1 = 16$

Dimensions are 5 ft by 16 ft.

Addition / Elimination Method:

$$\begin{cases} x + y = 4 \\ x - y = 6 \end{cases}$$

$2x = 10$

$x = 5$

$x + y = 4$

$5 + y = 4$

$y = 4 - 5$

$y = -1 \Rightarrow (5, -1)$

Consistent, Independent

Solve by addition/elimination method:

$$\begin{cases} 3x + 2y = 14 \\ 2(x - y = 3) \end{cases} \Rightarrow \begin{cases} 3x + 2y = 14 \\ 2x - 2y = 6 \end{cases}$$

$$5x = 20$$

$x = 4$

$3x + 2y = 14$

$3(4) + 2y = 14$

$12 + 2y = 14$

$2y = 2$

$y = 1$

Final Ans.

$(4, 1)$

System: Consistent

Eqns: Independent

Solve by addition/elimination method:

$$\begin{cases} 2x + 3y = 5 \\ -3(4x + y = -5) \end{cases}$$

$4x + y = -5$

$$\begin{cases} 2x + 3y = 5 \\ -12x - 3y = 15 \end{cases}$$

$$-10x = 20$$

$x = -2$

$4(-2) + y = -5$

$-8 + y = -5$

$y = -5 + 8$

$y = 3$

$(-2, 3)$

$\{(-2, 3)\}$

Solve by addition/elimination method:

$$3\begin{cases} 3x - 5y = 10 \\ 5\begin{cases} 2x + 3y = -6 \end{cases} \end{cases} \Rightarrow \begin{cases} 9x - 15y = 30 \\ 10x + 15y = -30 \end{cases}$$

$$19x = 0$$

$$x = \frac{0}{19} \qquad \boxed{x = 0}$$

$3(0) - 5y = 10$

$0 - 5y = 10$

$\boxed{y = -2}$

$(0, -2)$

The sum of two numbers is 28. 3 times one of them reduced by the other one is -7. Use system of linear equations in two variables to find both numbers.

Let x & y be the two numbers,

$$\begin{cases} x + y = 28 \\ 3x - y = -7 \end{cases}$$

$4x = 21$

$\boxed{x = \frac{21}{4}}$

$\boxed{x} + y = 28$

$\frac{21}{4} + y = 28$

LCD = 4

$21 + 4y = 112$

$4y = 91$

$\boxed{y = \frac{91}{4}}$

The numbers are $\frac{21}{4}$ & $\frac{91}{4}$.

There were 20 people in a meeting. The difference of twice the number of males and 3 times the number of females was equal to zero. use system of linear equations to find the number of males and the number of females in that meeting.

$$\begin{cases} M + F = 20 \\ 2M - 3F = 0 \end{cases} \Rightarrow \begin{cases} 3M + 3F = 60 \\ 2M - 3F = 0 \end{cases}$$

$5M = 60$

$M = 12$

$12 + F = 20$

$F = 8$

12 Males & 8 Females

Find eqn of a line that contains $(5, -3)$ and is parallel to $y = 3x + 4$. $m = 3$

Final Ans in Slope-Int. form.

$y = mx + b$

$y - y_1 = m(x - x_1)$

$y - -3 = 3(x - 5)$

$y + 3 = 3x - 15$

$y = 3x - 18$

Work on SG 10 & 11.